

Merits and Shortcomings of the Black-Scholes-Merton (BSM) Approach: An Analysis of the Role of Volatility

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Abstract

The historical background of option pricing dates back to 1900; before the Black-Scholes-Merton approach (BSM) was finalised, Louis Bachelier introduced the method of modelling option prices that was later extended by Sharpe and Lintner up until 1965 (Merton, 1973). With these foundations at hand, Black, Scholes and Merton finalised the model in 1973.

They constructed a model to determine the equilibrium value of an option by knowing the current price of the stock, the option striking price, the short-term interest rate (risk free), time to expiration (maturity), and the volatility of the return on the stock (Black and Scholes, 1971). The aim of this paper is to assess its economic and empirical validity as opposed to its shortcomings and limitations.

The paper is structured as follows. The first section describes the main assumptions of the BSM model. The second section provides an overview of the main merits and weaknesses of the approach, followed by a description of the contingent applications of the BSM in the next section. The final section is aimed at reminding the reader of the main points of the paper and provides constructive conclusions.

Keywords: Black-Scholes-Merton, BSM Model, Equilibrium Value

Assumptions of the BSM Approach

Before proceeding with the description of the fundamental assumptions of the BSM model on option pricing, it is necessary to define what an option actually is. As Black and Scholes (1973) assert, an option is a financial contract that gives an investor the right to buy or sell an asset within a specified time window and under determined conditions. There are two main kinds of options: the European option and the American option; the former differs from the latter as it gives the owner the right to sell the shares at a pre-arranged price, the exercise price (or strike price), at the pre-arranged date: the expiration date. On the other hand, the American option can be exercised at any date before maturity (Merton, 1977). Furthermore, option types can be divided according to flexibility in terms of execution: the owner of a call or a put option – namely options that can be bought or sold on or before a specific date - has the right to exercise it for a price said to be the option premium. Both of these two options can be either naked – when the owner does not own the underlying stock - or covered (Fortune, 1996). Nevertheless, the emphasis of the BSM is put on the European-style options.

The assumptions that make the BSM valid are presented below, amongst the standard assumptions (Bailey, 2005; Merton, 1998; Black and Scholes, 1971) for general option pricing models:

1. Markets are frictionless; there are no transaction costs in buying or selling an underlying asset, no penalties are imposed for short selling and no institutional restrictions are enforced on trading.
2. It is possible to lend and borrow in unlimited amounts, if a constant and risk-free interest rate is guaranteed. The assets are perfectly divisible, so that each part can be sold or lent independently.
3. The asset pays no dividends or other distributions during the life of the option, which is protected against possible stock splits. Additionally, the options considered are only the European type, meaning they can only be exercised at the pre-arranged maturity date.
4. There are no riskless arbitrage opportunities: this can be viewed more like an implication from the other assumptions rather than an assumption itself.

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5. Investors' preferences are assumed to be such that they would prefer to hold more rather than less.
6. Logarithmic returns of an underlying stock are normally distributed, and follow a continuous random walk according to a Brownian motion¹. Moreover, markets are continuously open so that trading can happen anytime.

With the last assumption, Black, Scholes and Merton made a significant contribution to the standard assumptions already existing in financial models such as the ones described by Sharpe (1964) and Lintner (1965) for the Capital Asset Pricing Model. The set of assumptions helps give credibility to the predictions. Bearing in mind the above-mentioned theorizations, it is now possible to outlay the formulae for the option premium (Hull, 2011) regarding European call (1) and put options (2).

$$1. \quad c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

$$2. \quad p = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$

where

$$3. \quad d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$4. \quad d_2 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

In particular, c and p are the European call and put option respectively; S_0 represents the initial stock price at time $t=0$ while K equals the strike price. R is the risk free rate, which is continuously compounded; T is the exercise or maturity date, and σ is the volatility of the stock price. $N(x)$ represents the cumulative probability distribution function for a $N(0,1)$ distribution; thus the probability that a variable that has a standard normal distribution² will be less than x .

¹ Returns follow a random path

² with a mean of zero and a standard deviation of one

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The empirical estimation given by Black and Scholes (1971) aimed at determining the equilibrium value of an option shows ambiguous results; the difficulty in estimating the variance caused the model to predict over-priced options on high variance stocks, and under-priced options on low variance stocks. The study also sheds light on the presence of non-constant variance, and the difficulty in completely ruling out the transaction costs that seem to be quite high. As can be seen from (1) and (2), the only variable that is impossible to observe before the option price is computed is the (explicit) volatility σ . Moreover, it is clear that the price of a call or put option does not depend on the expected rate of return, suggesting that even if investors do not agree on the return of some asset, the model will still hold. It is also independent of investors' beliefs and preferences, but all investors must agree on the value of the volatility for the model to be true.

Merits and Critiques of the Model

Although the model was constructed to be true only for the European-style options, Merton (1973) describes how if there are no dividend payments during the time up to the maturity date of an option - and hence there is no profit in exercising the option before its exercise date - then the BSM model applies also to American call options, that do not pay a dividend prior to the expiration date. In his paper, Merton underlines the versatility of the model and also expands on the arbitrage principle and dynamic hedging.

In particular, he explains how it is possible to minimise or totally eliminate the risk associated when an option is issued by constructing a replicating portfolio. Specifically, the financial intermediary creates a hedging portfolio made of risky and risk-free assets; if the payoffs of an option are exactly replicated by the portfolio, this is called a "replicating portfolio" (Merton, 1998, p. 329). When this happens, the replicating portfolio only consists of the underlying asset, which the call option was sold on.

Nevertheless, the dynamic hedging strategies are only possible theoretically; but when the hedging portfolio is well diversified, the risk involved purely concerns market risk - that cannot be hedged. This means the elimination of non-systematic risk, and only the market risk affects the return of the hedging portfolio. The adaptability of the model makes it possible for it to still be employed

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today to compute prices of put and call options; its simplicity is a crucial element that determined its success, alongside the fact that it provides a benchmark to assess competing models. It can also be used to price various elements of a firm capital's structure; in other words, the total value of the firm can be employed as an individual security thus replacing the concept of option under which the whole model is based (Merton, 1973).

However, the BSM approach has been shown to have several limitations too.

Recalling the Merton (1973) development of the dynamic hedging portfolio, making use of this strategy cannot always be possible due to discontinuous trading and transaction costs that violate assumption (1); additionally dynamic hedging could imply risk and is associated with costs, hence the risk-free implication does not hold.

Another of the assumptions can be subject to criticism; the perfect divisibility of assets (assumption 2, which allows for the assets to be split and sold or lent separately) is violated as often, financial intermediaries construct a bundle of option contracts to be traded. Moreover, assumption (6) can be argued: markets cannot allow continuous trading as there will always be execution lags – although these lags have been considerably reduced by the advent of new technologies that allow trading to happen within nanoseconds. Additionally, the formulae to get the price of a call and put option also lack an important element: Garleanu et al. (2009) show how the price of the option is determined not only by the factors shown in the BSM model, but also by its demand.

Fortune (1996) empirically confutes parts of the model assumptions, specifically the one regarding the normal logarithmic distribution of the underlying stock price, by observing that the relative frequency distribution is not normally distributed, is very leptokurtic (distribution whose points form a higher peak than in the normal distribution), and shows skewness.

Nonetheless, amongst the shortcomings of the model, particular importance must be given to the role of volatility. Although Black and Scholes (1973) were well aware of the difficulties that could be encountered when trying to estimate volatility, a wrong calculation could lead to completely wrong predictions of the option price. In order to give a well-rounded analysis it is necessary to

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first make a distinction between explicit (realised) and implicit (or implied) volatility. As Bailey (2005) states, the former is calculated as the standard deviation of previous data on asset price, with the following formula:

$$\sigma^2 = \frac{(g_1 - g)^2 + (g_2 - g)^2 + (g_3 - g)^2 + \dots + (g_N - g)^2}{N-1}$$

Where g is the sample average, $g_{1,2,3}$ etc. are the rates of return for each date. It is assumed that the volatility remains constant within time and does not depend on investor sentiment, although there is no consensus about how it should be modelled if it were non-constant.

Interestingly, implicit volatility is the market's assessment of the underlying asset's volatility – the market's estimate of the constant volatility parameter (Mayhew, 1995) – and the value that satisfies the BSM formulae. In order to calculate it, one would simply substitute the price of the option into the formula and use the volatility as the unknown to be found. One useful trait of implicit volatility is that it can be defined as forward looking, meaning that it is founded on the future expectations of the investors. Moreover, implicit volatility can be computed for different option contracts within the same underlying asset: when they are not equal, there is evidence of non-constant volatility and the BSM formulae do not hold (Bailey, 2005). In fact, implied volatility can graphically assume different shapes; as Mayhew (1995) underlines, across the strike price value they take the form of “smiles” or “skews”. Considering the defects of volatility, model-free estimates of the implicit volatility have been implemented; one of the most significant is the Chicago Board Options Exchange VIX index. Looking at the historical background, in 1993 the CBOE³ introduced the VIX index that was originally constructed to measure the market's expectations on the S&P100's (Standard and Poor's) 30-day implicit volatility and consequently, became the benchmark for the U.S. volatility. It is founded on the assumption of arbitrage absence and lack of discrete jumps. A derivation of the formula goes beyond the scope of this paper, but it takes the following form:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2$$

Figure 1 Derivation of Chicago Board Options Exchange VIX index

³ The CBOE Volatility Index – VIX, 2014

Where:

$\sigma^2 = (\text{VIX}/100)$; T is the time to expiration; F is the forward index level desired; K_0 equals the first strike below F ; K_i is the strike price of the i th out of the money option; ΔK_i represents the interval between strike prices; R equals the risk-free interest rate; $Q(K_i)$ is the “midpoint” of the bid-ask spread for each option with strike K_i . The VIX index can be considered as a market forecast of variability, because it reflects the beliefs and option preferences of the investors.

It can also be employed for other scopes: it constructs the underlying asset for options traded in the CBOE; it prices variance swaps – a type of forward contract; it estimates variance risk premia.

With these concepts about volatility in mind, Bollerslev and Zhou (2007) construct a different model to account for time changing volatility to explain price variation; they focus on the difference between implied and realised variances finding more accurate predictions than the ones of the BSM model, suggesting that the approach can be further implemented.

Alternative Applications

Although the Black-Scholes-Merton model was initially developed to fit the analysis of option prices, it can also be applied to other contingent claims and extended to the analysis of other types of derivatives. In particular, the analysis can be applied to any security whose payoff is dependent on the returns of another asset (Bailey, 2005).

Merton (1977, 1998) proceeds on extending the analysis and applies the BSM model in special cases, for example to loan guarantees and deposit insurance. Specifically, he claims that a contract that insures against the losses caused by the default of an underlying asset is equivalent to a put option whose exercise price is equal to the default-free contract value. Thus, a purchase of a debt instrument that involves possible defaults can be seen as an issue of a loan guarantee to insure the buyer against losses. Similarly, deposit insurance guarantees the depositor a refund (full or partial) in case of a bank run. Merton applies the analysis to the U.S. financial system and government guarantees; the contrast with the put option is made clear if we consider the deposit as an act of

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protection from the buyer against default, and the loan guarantee as a condition in which the bank holds the deposit and will only exercise if the borrower does not make the pre-arranged payments.

Additionally, Merton (1977) extends the analysis to non-financial instruments: the so-called “real-options”. These options regard real estate investments or development decisions, or, in general, every instrument that involves future uncertainty.

The BSM model can also be applied to revolving credit agreements, for which a company is obliged to give funds to another company in case this is in need of credit. The price of this decision is studied by Hawkins (1982) and depends on various elements such as the cost a bank might impose for borrowing (that could be less or more than the one imposed by the company lending the funds), and the rate at which funds are lent. Finally, Lauterbach and Schultz (1990) make use of the BSM (adjusted for dilution) to analyse price warrants rather than options, although their findings might suggest that the model performs rather poorly in this case.

Conclusion

The power of the Black-Scholes-Merton approach comes from the fact that it is a simple and straightforward way to calculate option prices, by using both explicit and implicit information in the formulae described earlier in the paper. Its versatility makes it possible for it to be applied to other contingent claims that are not strictly put and call options, as Merton (1977), Hawkins (1982) and Lauterbach and Schultz (1990) describe. For these reasons, the model is still widely used in the financial world today.

Nevertheless, the assumptions of the model often prove to be unrealistic; the fact that the volatility is not observable and the violations of other assumptions, for example the ones concerning continuous trading or perfect divisibility of an asset, prevent the model from being an accurate prediction of an option’s price. Thus, although the model is extremely effective in some aspects, it shall not be viewed as a flawless way of evaluating options’ and other security-like instruments’ prices; caution should be exercised when making empirical use of the approach.

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